Identification and Estimation of Semi-nonparametric Gravity Equation

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Introduction

- Many models in international trade assumes Constant Elasticity of Substitution (CES) utility function.
- CES is a key ingredient in deriving closed form expressions in these models.
- CES functional form is very restrictive.
- We propose a simple model of international trade with a more "flexible" utility function.
- We study Identification and Estimation of a Semi-nonparametric Gravity Equation.
- We propose a statistical test of CES.

CES and Trade Models

- CES looks something like this:

$$U(q_1, \ldots, q_N) = \left(\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \ (\sigma > 1).$$

- It is very tractable mathematically and numerically.
- Quantitative Trade Models (QTM) builds on having a CES aggregator:
  - Anderson & van Wincoop (2003), Eaton & Kortum (2002), Melitz (2003): Over 30k citations.
  - Made **Quantitative Analysis/Data Work** possible.
- Tractability helps deriving the Gravity Equation, a prediction of

$$\text{Tradeflow}_{i \to j} = \mathcal{M}(\text{Variables}, \text{Parameters}),$$

for some mapping $\mathcal{M}$ given by the model.

Cons of CES

- The elasticity of substitution σ governs all things welfare. (Welfare Predictions)
- Goods are either gross substitutes or gross compliments.
- The relative demand of two goods q_1 / q_2 is independent of the price of the third good. (IIA)
- The demand for any good remains st. positive no matter the price. (Missing Trade)
- CES + Monopolistic Competition fixes the mark up rate constant. (Marshall's 2nd law of demand)
- For a more extensive list, see Matsuyama (2023).
- Do we really want to restrict all of the above simultaneously ?

What we do

1. Matsuyama and Ushchev (2017) propose a nonparametric relaxation of CES that maintains both theoretical and numerical tractability.
2. They call this class the Homothetic with Single Aggregator (HSA) demand system.
3. We provide a numerical implementation of HSA demand.
4. HSA leads to a seminonparametric gravity equation.
5. We establish identification and the large-sample properties of its estimator, develop a test of CES, and establish the large-sample properties of counterfactual estimators.

Literature on HSA (Non-Gravity)

1. Theoretical Analysis with HSA
   - Matsuyama and Ushchev (2014, 2017, 2020, 2021, 2022), Grossman, Helpman and Lhuillier (2023), Matsuyama and Fujiwara (2026)
2. Numerical Analysis using HSA
   - Baqaee, Farhi, and Sangani (2024) : **Calibration** of HSA + Melitz Model.
3. Econometric Analysis with HSA
   - Chow, Kasahara, and Sugita (2026) identify production functions from firm revenue data using restrictions on demand under monopolistic competition. Their demand class includes the CoPaTh subclass of HSA.

We have an **estimator** for the demand system under **General Equilibrium**, whose large sample properties are studied.

Literature on Semi-nonparametric Gravity Equation

- Only one paper exists on Semi-nonparametric Gravity Equation.
- Adão, Arkolakis, and Ganapati (2020,Accepted JPE 2026Feb) studies a semi-nonparametric Gravity Equation.
  - Their model of the supply side is "flexible",
  - The demand side is CES,
- We have a flexible demand side (HSA) with a fixed supply.
- Combined with their work, trade economists can treat both the demand and the supply side nonparametrically.

Contribution

- We provide trade economists with a way to **estimate** the HSA demand system.
- This allows practitioners to **quantify** the effects of various policies under HSA.
- The popular CES specification is now testable, offering robustness.
- Large sample properties of some counterfactual analysis is also provided.

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Roadmap of the Section

1. Introduce CES and HSA gravity in an Armington economy.
2. Show how utility maximization and market clearing link all entries of the bilateral trade matrix.
3. Explain identification with known trade costs and no measurement error.
4. Introduce repeated equilibrium observations and the population objective that identifies the unknown share function.

CES gravity in an Armington economy

- There are N countries. Country i is endowed with ℓ_i units of its own variety.
- Under CES demand, the value of exports from i to j is

$$X_{i\to j} = \underbrace{p_j \ell_j}_{\text{expenditure in } j} \underbrace{\left(\frac{p_i \tau_{i\to j}}{A_j^{CES}(p)} \right)^{1-\sigma_0}}_{\text{share spent on variety } i}, \qquad \sigma_0 > 1,$$

where

$$A_j^{CES}(p) = \left[\sum_{k=1}^{N} (p_k \tau_{k\to j})^{1-\sigma_0} \right]^{1/(1-\sigma_0)}.$$

- p_i is the producer price and $\tau_{i\to j}$ is the iceberg trade cost.

Anderson and van Wincoop (2003). Equal taste weights for exposition.

HSA gravity in an Armington economy

- Replace the CES power function with an unknown share function S_{0i}.
- The value of exports from i to j becomes

$$X_{i\to j} = \underbrace{p_j \ell_j}_{\text{expenditure in } j} \underbrace{S_{0i}\left(\frac{p_i \tau_{i\to j}}{A_j}\right)}_{\text{share spent on variety } i}.$$

- The aggregator A_j is determined implicitly by adding up

$$\sum_{k=1}^{N} S_{0k}\left(\frac{p_k \tau_{k\to j}}{A_j}\right) = 1.$$

- Setting $S_{0i}(z) = z^{1-\sigma_0}$ gives the CES model on the previous slide.

Matsuyama and Ushchev (2017).

CD–CES mixture Other examples

General Equilibrium Structure

1. **Utility maximization through the adding-up constraint**
   The aggregator A_j ensures that expenditure shares sum to one

$$\sum_{k=1}^{N} S_{0k}\left(\frac{p_k \tau_{k\to j}}{A_j}\right) = 1, \qquad \sum_{k=1}^{N} X_{k\to j} = p_j \ell_j.$$

2. **Market clearing through the excess-demand function**

$$Z_i(p) = \underbrace{\frac{1}{p_i}\sum_{j=1}^{N} p_j \ell_j S_{0i}\left(\frac{p_i \tau_{i\to j}}{A_j}\right)}_{\text{total demand, including iceberg costs}} - \underbrace{\ell_i}_{\text{endowment}}.$$

Equilibrium prices satisfy $Z_i(p) = 0$ for every i, with $p_i > 0$ and $\sum_i p_i = 1$.

The equilibrium export matrix

- **Total spending equals total sales**

$$\underbrace{\sum_{k=1}^{N} X_{k\to i}}_{\text{row sum, total spending}} = p_i^* \ell_i = \underbrace{\sum_{j=1}^{N} X_{i\to j}}_{\text{column sum, total sales}} .$$

  Here, p^* is an equilibrium price vector.
- At the equilibrium price vector p^*, all bilateral exports are determined simultaneously. This produces the export matrix X:

$$\text{Destination} \begin{pmatrix} X_{1\to 1} & X_{2\to 1} & \cdots & X_{N\to 1} \\ X_{1\to 2} & X_{2\to 2} & \cdots & X_{N\to 2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1\to N} & X_{2\to N} & \cdots & X_{N\to N} \end{pmatrix} \quad \text{Exporter}$$

- Every entry obeys a destination's adding-up constraint (row) and an exporter's market-clearing condition (column).

What we observe

- We observe bilateral expenditure shares with measurement error

$$\lambda_{i\to j} = \frac{X_{i\to j}}{p_j^* \ell_j} + \varepsilon_{i\to j}$$

$$= \underbrace{S_{0i}\left(\frac{p_i^* \tau_{i\to j}}{A_j}\right)}_{\text{HSA expenditure share}} + \underbrace{\varepsilon_{i\to j}}_{\text{measurement error}} .$$

- The denominator $p_j^* \ell_j$ is destination j's total expenditure.
- Prices are recovered from observed expenditure and endowments. The model implies $E_i = p_i^* L_i$, hence $p_i^* = E_i / L_i$, followed by the chosen price normalization.
- We repeatedly observe the complete collection of bilateral shares $\{\lambda_{i\to j}\}_{i,j=1}^{N}$ and the equilibrium price vector p^*.
- Our objects of interest are the unknown share functions $S_{01}, \ldots, S_{0N}$.

Identification intuition

- For intuition, suppose trade costs $\tau_{i\to j}$ are known, there is no measurement error, and all goods share the same strictly decreasing function S_0.
- Let $z_{i\to j} = p_i^* \tau_{i\to j} / A_j$. Then

$$\lambda_{i\to j} = S_0(z_{i\to j}) \quad \Longrightarrow \quad S_0^{-1}(\lambda_{i\to j}) = \frac{p_i^* \tau_{i\to j}}{A_j}.$$

- The same A_j applies to every exporter supplying destination j. Hence, for any two exporters i and k,

$$\frac{S_0^{-1}(\lambda_{i\to j})}{S_0^{-1}(\lambda_{k\to j})} = \frac{p_i^* \tau_{i\to j}}{p_k^* \tau_{k\to j}}.$$

- In one realization, the known right-hand side restricts the inverse share function at the observed shares.
- Variation in equilibrium prices generates these restrictions at different share values. Under the joint-support conditions, they identify the common share function over an interval, up to normalization.

From noiseless shares to observed data

- Let (r) index repeated equilibrium observations. With measurement error, the observed share in observation (r) is

$$\lambda_{i\to j,(r)} = S_0\left(z_{i\to j}(P^*_{(r)}, S_0)\right) + \varepsilon_{i\to j,(r)}.$$

- Directly inverting an observed share would also invert its measurement error, rather than recover the true relative price.
- We rely on observing equilibrium outcomes repeatedly to smooth out the measurement errors.
- Trade costs are usually unknown. Our framework can accommodate this as well.

Unknown trade costs

Sampling repeated equilibrium outcomes

- Let (r) denote the rth repeated observation of the equilibrium.
- We observe the random sample

$$\left\{\left(\Lambda_{(r)}, P^*_{(r)}\right)\right\}_{r=1}^{R},$$

where the matrix of observed bilateral shares is

$$\Lambda_{(r)} = \begin{pmatrix} \lambda_{1\to 1,(r)} & \cdots & \lambda_{N\to 1,(r)} \\ \vdots & \ddots & \vdots \\ \lambda_{1\to N,(r)} & \cdots & \lambda_{N\to N,(r)} \end{pmatrix}.$$

The equilibrium price vector is $P^*_{(r)} = (P^*_{1,(r)}, \ldots, P^*_{N,(r)})'$.

- The observations are independent and identically distributed. The joint distribution of $(\Lambda_{(r)}, P^*_{(r)})$ remains fixed across r.

The population objective identifies demand

- Continue with known trade costs and a common share function. For a candidate S, define the population objective

$$Q(S) = \mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\lambda_{i\to j,(r)} - S\left(\frac{P^*_{i,(r)}\tau_{i\to j}}{A_j(P^*_{(r)}, S)}\right)\right\}^2\right].$$

- For every candidate, $A_j(P^*_{(r)}, S)$ is computed from the adding-up constraint. One candidate must explain all bilateral shares jointly.
- Under the support, strict-monotonicity, and joint-support conditions, and the normalization $A_N(\bar{p}, S) = 1$, the objective is uniquely minimized at the true share function S_0 on the identified support.

The statement holds without measurement error and also with conditionally mean-zero measurement errors of finite variance.

Comments

1. **The objective uses all bilateral shares.**
   General equilibrium links every entry of the latent share matrix through utility maximization and market clearing. The objective uses all entries of the observed matrix Λ jointly to exploit these model-implied restrictions.

2. At a single equilibrium price vector, the same candidate demand system must explain every bilateral share simultaneously.

3. We hold the number of countries N fixed and let the number of equilibrium observations R increase.

Why hold N fixed?

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Approximating the unknown share function

- Continue with known trade costs and a common unknown share function S_0.
- Approximate S_0 using an inverse-polynomial sieve

$$s_\kappa(z) = \sum_{q=0}^{\kappa} a_q z^{-q}.$$

- Let $\mathcal{S}_\kappa^*$ denote the admissible sieve functions satisfying the shape restrictions and the scale normalization

$$A_N(\bar{p}, s_\kappa) = 1.$$

- For each fixed κ, estimation searches over finitely many coefficients. Increasing κ allows a more flexible share function.

The sieve parameter space

- Write $s_\kappa(z) = \sum_{q=0}^{\kappa} a_q z^{-q}$ and $a = (a_0, \ldots, a_\kappa)'$. The admissible sieve space is

$$\mathcal{S}_\kappa = \left\{ s_\kappa \in \mathcal{S} \;\middle|\; \begin{array}{cc} \|a\|_1 \leq \rho_R^U, & \inf_{z \in K} s_\kappa(z) \geq \rho_R^L, \\ \multicolumn{2}{c}{\inf_{z \in K} \{-s_\kappa'(z)\} \geq \rho_R^L} \end{array} \right\}.$$

- K is a compact set explained on the next slide.
- Here, $\mathcal{S}$ imposes conditions sufficient for the unique existence of general equilibrium.
- Impose the scale normalization

$$\mathcal{S}_\kappa^* = \{ s_\kappa \in \mathcal{S}_\kappa \mid A_N(\bar{p}, s_\kappa) = 1 \}.$$

- The tuning parameters are κ, ρ_R^U, and ρ_R^L. Choose them so that $\mathcal{S}_\kappa^*$ is nonempty.

Why these restrictions matter

1. **The single aggregator is well defined.**
   For every equilibrium price vector p in the support and every admissible candidate s_κ, there is a unique $A_j(p, s_\kappa) > 0$ satisfying

$$\sum_{k=1}^{N} s_\kappa \left(\frac{p_k \tau_{k \to j}}{A_j(p, s_\kappa)} \right) = 1.$$

2. **Relative prices remain in a common compact set.**
   Under the maintained compact-support and uniform crossing conditions,

$$z_{i \to j}(p, s_\kappa) \equiv \frac{p_i \tau_{i \to j}}{A_j(p, s_\kappa)} \in K \subset \mathbb{R}_{++},$$

   uniformly over exporters, destinations, equilibrium prices in the support, and admissible candidates.

   These properties make predicted shares well defined and allow uniform bounds in the large-sample analysis.

The sieve least-squares estimator

- Replace the population expectation with an average over the R observed equilibrium outcomes

$$\widehat{Q}_R(s_\kappa) = \frac{1}{R}\sum_{r=1}^{R}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\lambda_{i\to j,(r)} - s_\kappa\left(\frac{P^*_{i,(r)}\tau_{i\to j}}{A_j(P^*_{(r)}, s_\kappa)}\right)\right\}^2.$$

- The sieve estimator is

$$\widehat{S}_\kappa \in \underset{s_\kappa \in \mathcal{S}^*_\kappa}{\arg\min}\ \widehat{Q}_R(s_\kappa).$$

- For every candidate s_κ, recompute the aggregators for each observation and construct all bilateral shares jointly.
- Although the sieve is linear in its coefficients, the criterion is nonlinear because the aggregators depend on s_κ.

Weak consistency of the sieve estimator

- Write the candidate share as

$$\widehat{\lambda}_{i\to j}(p,s) = s\left(\frac{p_i\tau_{i\to j}}{A_j(p,s)}\right).$$

- Measure the distance from the truth by

$$M^2(s,S_0) = \sum_{i=1}^{N}\sum_{j=1}^{N}\int\left[\widehat{\lambda}_{i\to j}(p,s) - \widehat{\lambda}_{i\to j}(p,S_0)\right]^2\mu(dp),$$

where μ is the distribution of the equilibrium price vector.

Theorem — Weak consistency

Under the sampling, identification, and sieve regularity conditions, there exists a sequence of tuning parameters such that, with N fixed and $R\to\infty$,

$$M(\widehat{S}_{\kappa_R}, S_0) \xrightarrow{p} 0.$$

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Roadmap

1. Test of the CES model
2. Consistency of the counterfactual analysis

The Matsuyama–Ushchev test

- Can we test the CES restriction within HSA?

$$H_0 \colon S_0(z) = z^{1-\sigma_0} \quad \text{for some } \sigma_0 > 1.$$

- For a fixed, known $c > 0$, define

$$\Phi_j(p, s) = \sum_{i=1}^{N} \int_c^{z_{i\to j}(p,s)} \frac{s(z)}{z}\, dz.$$

- The characterization of Matsuyama and Ushchev (2017, Proposition 1, Assertion (iii)) implies that, under CES,

$$\Phi_j(p, S_0) = \Gamma(c, \sigma_0) \equiv \frac{1 - Nc^{1-\sigma_0}}{1 - \sigma_0}$$

  for every destination and equilibrium price vector in the support.
- We turn this implication into a scalar moment restriction.

Large-sample behavior of the test

- Let $\widehat{\sigma}$ be the CES estimate. The estimated discrepancy is

$$\widehat{D}_R = \frac{1}{RN} \sum_{r=1}^{R} \sum_{j=1}^{N} \Phi_j(P^*_{(r)}, \widehat{S}_\kappa) - \Gamma(c, \widehat{\sigma}).$$

- Under H_0 and the additional linearization, undersmoothing, and variance regularity conditions,

$$Z_R = \frac{\sqrt{R}\,\widehat{D}_R}{\sqrt{\widehat{V}_{T,\kappa}}} \xrightarrow{d} \mathcal{N}(0,1).$$

  Here, $\widehat{V}_{T,\kappa}$ consistently estimates the asymptotic variance, accounting for both HSA and CES estimation.
- Reject at the 5 percent significance level when $|Z_R| > 1.96$.
- The test is consistent against fixed alternatives with a nonzero population discrepancy. It need not detect every non-CES demand system.

Counterfactual welfare

- A researcher changes the trade-cost matrix from τ to τ^{CF}, holding the endowment vector ℓ fixed.
- Let p^0 and p^{CF} be the corresponding equilibrium prices. The change in log indirect utility for destination j is

$$\Delta_j^*(\ell) = \log \frac{p_j^{CF}\ell_j}{P_j^{I,CF}} - \log \frac{p_j^0\ell_j}{P_j^{I,0}}.$$

- The ideal price index is obtained from HSA demand through

$$\log P_j^I(p,s) = \log A_j(p,s) + \Phi_j(p,s),$$

  evaluated using the trade costs in the relevant scenario.
- Changing trade costs affects welfare through both equilibrium income and the cost of consumption.

Consistency of counterfactuals

- Evaluate the policy at observation 1. Estimate demand using observations $r = 2, \ldots, R$, obtaining $\widehat{S}_{\kappa,-1}$.
- Replace S_0 with $\widehat{S}_{\kappa,-1}$, solve for equilibrium prices under each scenario, and compute $\widehat{\Delta}_{j,(1)}$.

Counterfactual consistency

Under estimator consistency, continuity of the welfare and excess-demand functionals, and the maintained compactness and unique-equilibrium conditions for both scenarios,

$$\widehat{\Delta}_{j,(1)} - \Delta^*_{j,(1)} \xrightarrow{p} 0.$$

- The target $\Delta^*_{j,(1)} = \Delta^*_j(L_{(1)})$ is random because the held-out endowment vector is random.
- The estimated counterfactual therefore recovers the distribution of the unobserved true counterfactual in large samples.

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Simulation design

- Use the Cobb–Douglas–CES mixture with equal weights

$$S_0(z) = \frac{\eta_0}{N} + (1 - \eta_0) z^{1-\sigma_0}, \qquad \eta_0 \in \{0, 0.25, 0.5\}, \quad \sigma_0 = 2.$$

- $N = 27$ countries and 1,000 replications. Draw IID endowments, solve for equilibrium prices, and add measurement error to shares. The signal-to-noise ratio is approximately 2.
- Unlike the known-cost illustration, estimate trade costs jointly with demand using $\tau_{i \to j}(\theta) = \exp(W_{i \to j}\theta)$ for $i \neq j$, with $\theta_0 = 5$ and fixed bilateral distances $W_{i \to j}$.
- Compare CES with normalized HSA sieves $\kappa = 1, 2, 3$. The truth belongs to the $\kappa = 1$ sieve.
- Use $R = 20$ for estimation and counterfactuals, and $R = 20, 40, 60$ for the test.

Parameter recovery

Mean estimates of the trade-cost parameter, $\theta_0 = 5$

η_0	HSA $\kappa = 1$	HSA $\kappa = 2$	HSA $\kappa = 3$	CES
0	5.005	4.943	4.935	5.002
0.25	5.005	4.938	4.936	4.521
0.50	5.004	4.907	4.919	4.039

- HSA mean estimates lie within 0.1 of the truth across all designs and sieve dimensions.
- CES recovers the truth when correctly specified. Under the alternatives, its estimates move away from the structural parameter.
- The additional sieve coefficients have mean estimates smaller than 0.001 in magnitude.

Matsuyama–Ushchev test rejection rates

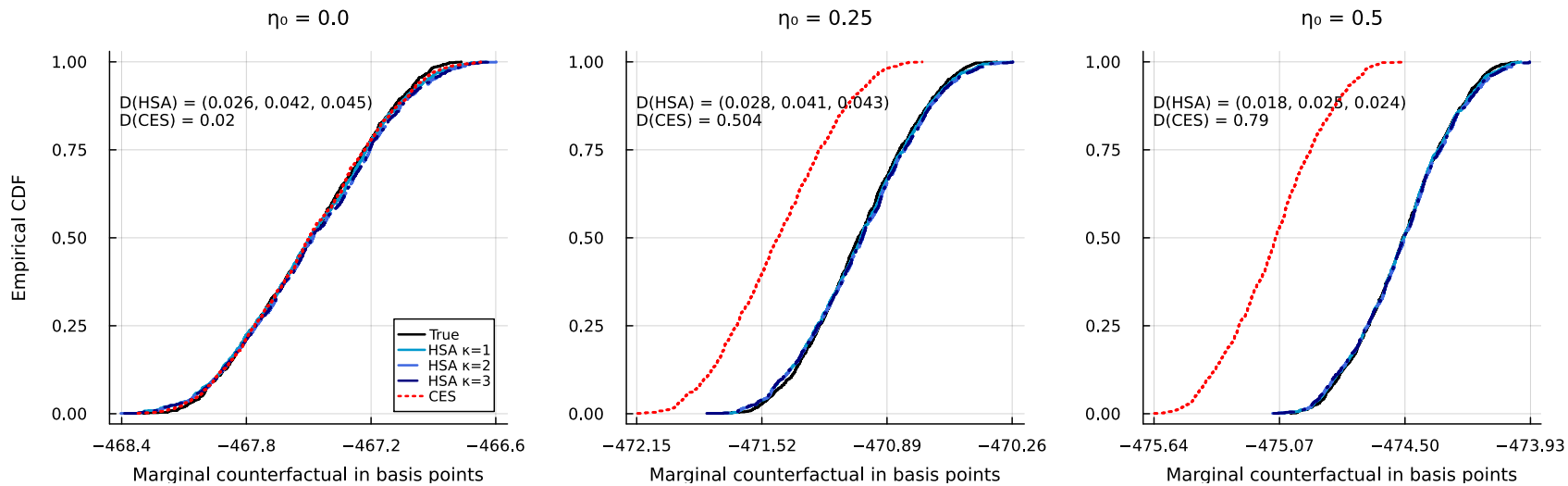
η_0	κ	$R = 20$	$R = 40$	$R = 60$
0	1	8.4%	6.4%	6.4%
0	2	8.1%	6.4%	6.6%
0	3	3.6%	2.3%	2.2%
0.25	1	100%	100%	100%
0.25	2	99.0%	100%	100%
0.25	3	98.0%	100%	100%
0.50	1	61.7%	84.9%	95.1%
0.50	2	34.0%	52.3%	69.6%
0.50	3	24.7%	41.4%	60.9%

Nominal level 5 percent. 1,000 replications. $c = 5$.

- Power improves with R in the $\eta_0 = 0.5$ design.
- The test is conservative under the null for $\kappa = 3$. Power need not increase with η_0 because the test uses a scalar moment.

Recovering the counterfactual distribution

Hold out observation 1 and estimate the marginal welfare effect of a 5 percent increase in foreign trade costs into destination N.



- HSA closely tracks the true distribution across all three sieve dimensions.
- CES tracks the truth under the null but deviates under misspecification.

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Conclusion

1. We study the identification and estimation of a seminonparametric gravity equation under the HSA demand system.
2. We establish the large-sample properties of the estimator of HSA demand under repeated-observation asymptotics.
3. We provide a framework for testing the CES model and frequentist guarantees for selected counterfactual analyses.
4. **Future work**
   Whether this framework can be extended beyond a simple endowment economy remains an open question.

7 Appendix

Unknown trade costs

- Known trade costs were a simplification for the identification intuition. In the full model, trade costs are unknown.
- Following the convention, we adopt a parametrization

$$\tau_{i\to j}(\theta_0) = \exp\left(W_{i\to j}^{\top}\theta_0\right), \qquad i \neq j, \qquad \tau_{i\to i} = 1.$$

- The vector $W_{i\to j}$ contains fixed, observed bilateral characteristics such as distance and a contiguity indicator.
- The finite-dimensional parameter θ_0 and the common share function S_0 are jointly unknown.
- We restrict θ to ensure that the implied trade costs satisfy the iceberg-cost restrictions.

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Why hold the number of countries fixed?

1. **Keep the object of interest fixed.**
   In an Armington economy, increasing N adds varieties to the consumer's utility maximization problem. The demand system being estimated therefore need not remain the same along $N \to \infty$.

2. **Keep the data-generating process well defined.**
   The usual finite-dimensional fixed-point argument establishes equilibrium existence for each fixed N, but does not by itself establish existence in the limiting economy. That requires a separate argument.

   We hold N fixed and increase the number of equilibrium observations R.

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Example — Cobb–Douglas–CES mixture

- Consider the share functions

$$S_{0i}(z) = \eta_0 \alpha_i + (1 - \eta_0) z^{1-\sigma_0}, \qquad 0 \le \eta_0 < 1, \quad \sigma_0 > 1,$$

  where $\alpha_i \ge 0$ and $\sum_i \alpha_i = 1$.
- The adding-up constraint gives the CES aggregator

$$A_j = \left[\sum_{k=1}^{N} (p_k \tau_{k \to j})^{1-\sigma_0} \right]^{1/(1-\sigma_0)}.$$

- Setting $\eta_0 = 0$ recovers CES.
- If $\eta_0 \alpha_i > 0$, the expenditure share on good i cannot fall below $\eta_0 \alpha_i$. The good is essential, even though consumers can substitute toward other goods.

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Other examples of HSA

- **Constant but different elasticities**

$$S_{0i}(z) = z^{1-\sigma_{0i}}, \qquad \sigma_{0i} > 1.$$

The elasticity of the share function with respect to its relative-price argument is constant for each good but may differ across goods

$$\frac{zS'_{0i}(z)}{S_{0i}(z)} = 1 - \sigma_{0i}.$$

- **Translog demand**
  Under suitable parameter restrictions, translog demand is also a parametric member of the HSA class. See Matsuyama (2023).
- HSA therefore accommodates several departures from CES, without requiring us to select one of these parametric forms.

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